NASA TT F-11,803

Page One Title

SLENDER TWO-DIMENSIONAL BODIES OF MINIMUM WAVE DRAG IN A NONEQUILIBRIUM SUPERSONIC FLOW

A. N. Krayko and R. A. Tkalenko

Translation of "Tonkiye Ploskiye Tela Minimal'nogo Volnovogo Soprotivleniya v Neravnovesnom Sverkhzvukovom"
Izvestiya Akademii Nauk SSSR, Mekhanika Zhidkosti i Gaza, No. 4, pp. 56-63, 1967

FACILITY FORM 602
N68-30388 (ACCESSION NUMBER)
14 (PAGES)
(NASA CR OR TMX OR AD NUMBER)
(THRU)
1 (CODE)
01 (CATEGORY)

AUG 1968
RECEIVED
NASA STI FACILITY
INPUT BRANCH

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 JULY 1968

SLENDER TWO-DIMENSIONAL BODIES OF MINIMUM WAVE DRAG IN A NONEQUILIBRIUM SUPERSONIC FLOW

A. N. Krayko and R. A. Tkalendo

ABSTRACT. The determination of two-dimensional, slender profiles of minimum drag in the case of nonequilibrium supersonic flow is carried out in linear approximation. This approach yields a more complete representation of the optimum profile form. The problem considered here consists in constructing a minimum drag contour passing through points a and c at a given free flow and a fixed maximum length of $L^\circ = x_c^\circ$, which is taken as the characteristic linear dimension. Computations by the iterative method were carried out on an M-20 computer. The results, presented in graphs and a table, show that the decrease in drag obtained with optimal profiles is insignificant when compared with contours made of rectilinear segments.

The problem of profiling optimum contours of two-dimensional and axisymmetric bodies in a supersonic, nonequilibrium flow without shock waves (including particularly the contours of tail ends and nozzles) was reduced in [1] to a boundary problem for a hyperbolic system of equations which includes the flow equations and equations for the Lagrange factors[1]. Since its solution is complicated, the construction of the optimum nozzle contour in [2] was accomplished on the basis of a linear approximation. Although this approach makes it possible to establish the order of the possible gain, any conclusion concerning the shape of the resulting contour is primarily of a qualitative nature. Below, the construction of slender bodies with minimum wave drag in a nonequilibrium supersonic flow is carried out in a linear approximation which leads to a more complete representation of the optimum contour shape. Many examples showing the application of the linear theory to the optimization of body shapes in the supersonic flow of an ideal gas are presented in [3]. /56[2]

1. Let x and y be rectangular coordinates relative to some characteristic linear dimension. The origin of the coordinates is placed at the frontal point a of the specified plane contour region while the x axis is directed along the velocity of the uniform incident flow which flows from the left to the right.

[1]There is an error in equation (4.5) of [1]: the minus sign should be placed in front of the third term within the braces.

[2]Numbers in the margin indicate pagination in the foreign text.

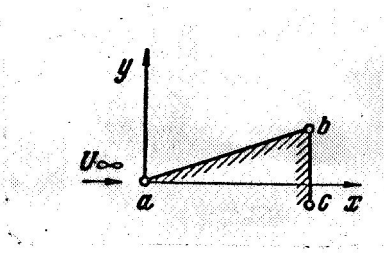

Figure 1.

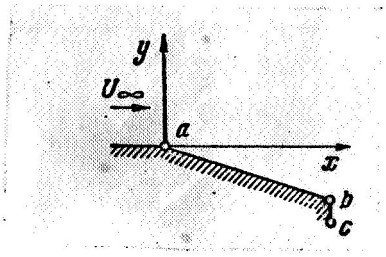

Figure 2.

The incident flow is assumed to be in equilibrium and to be supersonic such that $M_\infty > 1$, where M_∞ is the Mach number determined from the "frozen" velocity of sound and gas velocity U_∞°. The superscript is assigned to dimensional quantities while the subscript ∞ is assigned to the parameters of the unperturbed flow. The Mach number determined from U_∞° and from the equilibrium sound velocity is designated by Me_∞. Other subscripts will be assigned to parametric values for different points of the contour.

Our problem is to construct a contour ac passing through points a and c and providing for the minimum wave drag when the incident flow is known and the maximum fixed permissible length is $L^\circ = x_c^\circ$ and is assumed to be the characteristic linear dimension (Figures 1 and 2). It was shown in [4] that the optimum contour may include the region of an extremum when a limitation is imposed on the length. This extremum is the end bc, where $x \equiv 1$. Unlike region ab, the end is not in the gas flow and therefore the pressure p+ is the same at all of its points (the pressure is relative to $\rho_\infty^\circ U_\infty^{\circ 2}$, where ρ is the density).

We shall assume that the value of p+ is known and is independent of the shape of contour ac when the parameters of the incident flow are fixed. /57

We shall find the distribution of pressure p on ab by using the linear theory. Under these conditions we have the following expression for the pressure [5] on the surface of the body with generatrix $y = y(x)$

$$p(x) = p_\infty + \beta_\infty^{-1}\left[y'(x) + \int_0^x y'(t)\psi'(x-t)\,dt\right] \equiv$$

$$\equiv p_\infty + \beta_\infty^{-1}\frac{d}{dx}\int_0^x y'(t)\psi(x-t)\,dt \qquad (\beta_\infty = \sqrt{M_\infty^2 - 1}) \qquad (1.1)$$

In the above equation the primes designate derivatives with respect to the argument which controls the given function and $\psi(x)$ is a known function of x. The equation is valid when breaks are present or absent on the contour of the points.

The function $\psi(x)$ is determined by the properties of the gas and, in particular, by the number of nonequilibrium processes n. The physical and chemical constants together with L°, p_∞°, ρ_∞° and U_∞° are contained in $\psi(x)$ in the form of n(n+1) dimensionless combinations. The use of the Onsager relation reduces their number to 1/2n(n+3). In the case of a two-dimensional profile or body of rotation located in an infinite flow, $\psi(x)$ is continuous where $0 \leqslant x \leqslant 1$, is equal to unity when $x = 0$ and is equal to zero when $x < 0$. With flow in nozzles, $\psi(x)$ is continuous for $0 \leqslant x \leqslant 1$ only when $L^\circ/D^\circ < \beta_\infty$, where D° is the diameter or width of th initial nozzle cross section, i.e., for short nozzles, where a characteristic energing from point a does not reach the opposite wall of the nozzle.

For the same nonequilibrium process, the function $\psi(x)$ for the upper wall of a plane contour (or for the lower wall of a plane nozzle where $L^\circ/D^\circ < \beta_\infty$) has the form [6]

$$\psi(x) = F(x) + \frac{L^\circ}{K^\circ}\int_0^x F(\tau)\,d\tau$$

$$F(x) = I_0[(\omega-1)X]\exp[-(\omega+1)X]$$

$$X = \frac{L^\circ}{2K^\circ}x, \quad \omega = \frac{M_{e\infty}^2 - 1}{M_\infty^2 - 1} \geqslant 1 \qquad (1.2)$$

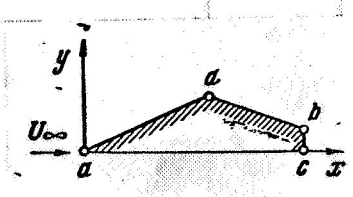

Figure 3.

In the above equations K° is a constant which has the dimensionality of length and defines the relaxation length. I_i is a modified Bessel function of the first generation and of the i-th order. In the same case, if the nonequilibrium processes which take place with a finite velocity are absent, then $\psi(x) \equiv 1$ when $x \geqslant 0$.

Let us introduce the function z(x) to ab in accordance with the equation

$$y' - z = 0 \qquad (1.3)$$

Then the condition under which the linear theory is applicable may be written in the form

NASA

$$|z| \leqslant k < \infty \text{ на } ab \tag{1.4}$$

where k is a known constant.

Additional restrictions are possible. For example the minimum permissible value of the ordinate Y (Figure 3) may be given for a certain $x = x_d < 1$, i.e.

$$y_d \equiv y(x_d) \geqslant Y \tag{1.5}$$

The wave drag χ is

$$\chi = \int_0^{x_b} \beta_\infty p y' \, dx + \beta_\infty p^+ (y_c - y_b) \tag{1.6}$$

with an accuracy determined by the trivial constant β_∞. Therefore we can formulate the following variational problem. We must find among the functions /58
y(x), z(x), and p(x) those which yield the minimum value for the functional (1.6), satisfy equations (1.1) and (1.3), and satisfy the condition that points a and c are traversed: Y(0) = 0 and $y(1) = y_c$. They must also satisfy inequality (1.4) or inequalities (1.4) and (1.5). Under these conditions $0 = x_a \leqslant x \leqslant x_b \leqslant x_c = 1$.

In the future we shall use the following equation in place of (1.6):

$$\chi = \beta_\infty p^+ y_c + \beta_\infty (p_\infty - p^+) y_b + \int_0^{x_b} y'(x) \frac{d}{dx} \int_0^x y'(t) \psi(x - t) \, dt \, dx \tag{1.7}$$

which is obtained from (1.6) and (1.1).

2. To solve the variational problem we formulate the functional

$$\Omega = \chi + \int_0^{x_b} \mu(x)(y' - z) \, dx$$

where χ is given by equation (1.7) while $\mu(x)$ is an indeterminate Lagrange factor. Because of (1.3) the variation of Ω and χ coincide for all permissible variations. Let us find the first variation of Ω.

Several factors should be considered in carrying out the variation. First of all the contour may have break points and points where the factor μ is discontinuous. We shall designate these points by d,d°,... and f,f°,... respectively (in the general case we may have several such points). If any values are discontinuous, then before or after the discontinuity point they will be assigned a minus or plus subscript, respectively. The difference between the values (variation) of the parameter $\xi(x)$ for the varied contour and the initial contour is designated by $\delta\zeta$ for any given x. The increments of coordinates x and y for the break points of the same contours are designated by Δx and Δy. We note that $\Delta y = \delta_{\pm} + y'_{\pm} \Delta x$ and that $\Delta x_a = \Delta y_a = \Delta y_e = 0$. Also, the order of integration is reversed so that when breaks are absent we have

$$\int_0^{x_b} y'(x)\frac{d}{dx}\int_0^x \delta y'(t)\psi(x-t)\,dt\,dx =$$

$$= y_b'\delta y_b + \int_0^{x_b}\left[y'(x)\psi'(0) + y''(x) + \int_x^{x_b} y'(t)\psi''(t-x)\,dt \right]\delta y(x)\,dx$$

By performing the calculation steps, including integration by parts, we obtain

$$\delta\chi \equiv \delta\Omega = [\beta_\infty(p-p^+)y']_b\,\Delta x_b + [\beta_\infty(p-p^+) + y' + \mu]_b\,\delta y_b +$$

$$+ \sum_{r=d,\,d^\circ,\ldots} [2(y_-' - y_+')\delta y_- + (y_-' - y_+')^2\Delta x]_r + \sum_{r=f,\,f^\circ,\ldots} (\mu_- - \mu_+)_r\,\delta y_r +$$

$$+ \left(\int_a^i + \ldots + \int_j^b\right)[(W-\mu')\delta y - \mu\delta z]\,dx \qquad (2.1)$$

Here summation is carried out over all points of the discontinuity; the integration interval is broken down into continuous sections y' and μ; i and j are the first and last points of the discontinuity; the function W is given by the equality

$$W = -2y''(x) - \int_0^x y'(t)\psi''(x-t)\,dt + \int_x^{x_b} y'(t)\psi''(t-x)\,dt$$

Since we are considering only slender bodies for which $|y'| \leqslant k$, the initial contours may consist of only two types of sections $|z| < k$ and

$$y' \equiv z = \pm k \qquad (2.2)$$

Since μ can be selected arbitrarily, we will change all coefficients in (2.1), in front of δy on the sections of (2.2) and the coefficients in front of /59

δz on the remaining sections to zero, i.e., we let

$$\mu' = W \text{ (where } z = \pm k), \quad \mu = \mu' = 0 \text{ (where } |z| < k) \tag{2.3}$$

As the conditions for the integration of (2.3) we shall require that

$$\mu_{r-} = 0 \quad (r = l, l^\circ, \ldots) \tag{2.4}$$

are satisfied at the right end of each section of (2.2), i.e., $l, l^\circ, \ldots$ are those points of $f, f^\circ, \ldots$, which are the right ends of these sections. If point b is the right end of such a section then (2.4) must also be satisfied in it. It is clear that $\mu(x)$ may be chosen as described for any contour regardless of whether it is an optimum one or not.

From (2.1), (2.3) and (2.4) we obtain

$$\delta\chi = -y_b'^2 \Delta x_b + [\beta_\infty (p - p^+) + y']_b \Delta y_b + \sum_{r=d,\, d^\circ, \ldots} [2(y_-' - y_+')\delta y_- + (y_-' - y_+')^2 \Delta x]_r + \sum_{r=j,\, j^\circ, \ldots} \mu_{r+} \delta y_r + \left(\int_i^m + \int_{i^\circ}^{m^\circ} + \ldots\right) W \delta y \, dx - \left(\int_j^l + \int_{j^\circ}^{l^\circ} + \ldots\right) \mu \delta z \, dx \tag{2.5}$$

Here im, $i^\circ m^\circ, \ldots$ are segments on which $|z| < k$, while jl, $j^\circ l^\circ, \ldots$ are segments of (2.2). Segments of the two types may alternate. Then the right ends of segments of one type are the left ends of the segments of the other type while points a and b are the left and right points of the first and last segments. Segments of the same type may be joined at the break points.

If the initial contour was optimum, then any permissible variation of such a contour must lead to an increase in χ, i.e., in this case $\delta\chi \geqslant 0$.

3. Let us first consider the problem without restriction (1.5). Here the signs of permissible variations of δy by im, $i^\circ m^\circ, \ldots, \Delta x_b$ for $x_b < 1$, Δy_b for $y_b \neq y_c$, δy_{r-} and Δx_r for $r = d, d^\circ \ldots$ and δy_r for $r = j, j^\circ, \ldots$ are arbitrary while Δx_b for $x_b = 1$, Δy_b for $y_b = y_c$ and δz for $y' = \pm k$ satisfy conditions $\Delta x_b \leqslant 0$, $\Delta y_b \geqslant 0$ and δz sign $z \leqslant 0$.

We can see from this and from (2.5) that contour ac has the maximum permissible length when $y'_b \neq 0$, i.e., $x_b = 1$, and when there are no limitations

on the minimum permissible thickness section ab will be smooth. In this case the condition that end bc be a section of the boundary extremum ($-y_b^{12} < 0$) is automatically satisfied. The shape of the regions of the two-sided extremum, where $|y'| < k$ are the values of ordinates of point b when $y_b > y_c$, and the length of the sections of the boundary extremum of (2.2) are determined by the equalities

$$W = 0, \quad \beta_\infty(p_b - p^+) + y_b' = 0, \quad \mu_{r+} = 0 \quad (r = j, j^\circ, \ldots) \tag{3.1}$$

where $j, j^\circ, \ldots$, are those points of $f, f^\circ, \ldots$, which are the left ends of segments of (2.2). The condition that segments of (2.2) are sections of the boundary extremum written in the form of the inequality

$$\mu(x)\operatorname{sign} y' \leqslant 0 \quad \text{where } y' = \pm k \tag{3.2}$$

If points b and c coincide then the following inequality must be satisfied (condition for the absence of the end) instead of the second condition of (3.1):

$$\beta_\infty(p_b - p^+) + y_b' \geqslant 0 \tag{3.3}$$

Inequality (3.3) must be satisfied in the case when $y_b \neq y_c$, but the entire contour consists of a single region of the boundary extremum $y' = -k$.

The second equality of (3.1) and inequality (3.3) represents the linear /60
formulation of the Busemann conditions. These conditions were first obtained to solve the exact formulation of variational problems of equilibrium gas dynamics [4, 7, 8]. Later [1, 9] it was shown that they were also valid for nonequilibrium flows. It follows from (3.1) and (3.3) that the contour shown in Figure 1 is realized when $p+ > p_b$, while the contour shown in Figure 2 is realized when $p+ < p_b$.

The above conditions, together with the equation

$$y(x) = \int_0^x y'(t)\,dt \tag{3.4}$$

completely define the form of the optimum contour of ac.

4. The first equation of (3.1) which gives the shape of the regions of the two-sided extremum taking into account the expression for W, the

substitution $x_b = 1$ and single-stage integration, may be rewritten in the form

$$2y'(x)+\int_0^x y'(t)\psi'(x-t)dt+\int_x^1 y'(t)\psi'(t-x)dt=C \tag{4.1}$$

If the initial point of the contour is not an element of section (2.2), then the integration constant C is given by the equality

$$C=2y_a'+\int_0^1 y'(t)\psi'(t)dt \tag{4.2}$$

If the nonequilibrium processes are absent, when $\psi(x) \equiv 1$, it follows from (4.1) that $y'(x) = \text{const}$, i.e., the section of the two-sided extremum is a straight line segment.

We will substitute the variable in (4.1) by

$$\tau=1-t \tag{4.3}$$

It is easy to see that

$$\int_0^{1-x} y'(t)\psi'(1-x-t)dt=\int_x^1 y'(1-\tau)\psi'(\tau-x)d\tau$$
$$\int_{1-x}^1 y'(t)\psi'(t-1+x)dt=\int_0^x y'(1-\tau)\psi'(x-\tau)d\tau \tag{4.4}$$

From the equalities we obtain an equation which can be used to determine $y'(1-x)$ and which is just like equation (4.1) used to determine $y'(x)$. Consequently the slope $y'(x)$ of the extremal contour, which consists of a single section of a two-sided extremum is symmetric with respect to the center of the interval ($x=0.5$). This conclusion is valid for any function $\psi(x)$, i.e., when there is an arbitrary number of nonequilibrium processes. Integration of (2.3) gives us

$$\mu(x)+2y'(x)+\int_0^x y'(t)\psi'(x-t)dt+\int_x y'(t)\psi'(t-x)dt=C \tag{4.5}$$

If we use (4.1), (4.5), (2.5) and the last condition of (3.1) we can show that in the case of an optimum contour which does not have break points on ab the constants C in (4.1) and (4.5) coincide and are also the same on all

regions of the two-sided and boundary extremum value. These equations and conditions, together with (4.3) and (4.4), can be used to show that when breaks are absent there is symmetry with respect to x = 0.5 just as in the case of a contour which consists only of a single region of a two-sided extremum value. This is true not only for y'(x), but also for μ(x), i.e. /61

$$y'(1-x)=y'(x), \qquad \mu(1-x)=\mu(x) \tag{4.6}$$

Finally, it follows from (4.1) and (4.2) that y'(x) either retains its sign everywhere when $0 \leqslant x \leqslant 1$ or $y'(x) \equiv 0$. Consequently $p(x) \equiv p_\infty$ and $p+ = p_\infty$. The length of the optimum contour can be less than the maximum possible value only in this special case and, as in the case of the linear consideration [2], there is an infinite number of optimum contours with the same drag.

Different optimum contours correspond to different values of p+ for a fixed incident flow, i.e., different functions y(x), y'(x) and drag coefficients of segment ab

$$c_x = \beta_\infty \int_0^1 (p-p_\infty) y' dx \tag{4.7}$$

If we take y_a' instead of p+ as the parameter which characterizes the contour, then y(x), y'(x), c(x) and p+, for which the given contour is optimum, will be functions of y_a'. In this case if $y_a' \neq 0$ and ab consists only of a section of the two-sided extremum then, as follows from (1.1), (1.6), (3.1), (3.4), (4.1), (4.2), and (4.7), sets

$$\frac{c_x}{y_a'^2}, \quad \frac{\beta_\infty (p_\infty - p^+)}{y_a'}, \quad \frac{y_b}{y_a'}, \quad \frac{y(x)}{y_a'}, \quad \frac{y'(x)}{y_a'} \tag{4.8}$$

are universal, i.e., they do not depend on y_a' but are determined only by the form of the function ψ(x) and, consequently, by those dimensionless constant (related to the property of the gas and the parameters of the unpeturbed flow) which are contained in this function. Consequently, in this case it is sufficient to construct a single optimum contour. Contours corresponding to other values of p+ (or y_a') are obtained from this contour by simple conversion.

5. The solution of the problem for the case when the minimum permissible thickness Y is given at some internal point $0 < x_d < 1$ is somewhat different from that considered above. If the ordinate y_d is determined from the solution of the same problem, but without the thickness limitations, and turns out

to be less than Y, then the optimum contour at the $x = x_{d'}$, $y = Y$ has a break (Figure 3). In this case we can use (2.6) to show that all permissible displacements of the break (which increase the thickness of the contour) lead to an increase in χ. There are no breaks on ad and db. Here, as before, relations (3.1) - (3.3) are satisfied. At the same time equation (4.1) should be replaced by

$$2y'(x)+\int_0^x y'(t)\psi'(x-t)\,dt+\int_x^1 y'(t)\psi'(t-x)\,dt=C_- \tag{5.1}$$

for $0 < x < xd$ and by an analagous equation with a constant C+ for $xd < x < 1$. The constants C- and C+ have the following values:

$$C_-=2y_a'+\int_0^1 y'(t)\psi'(t)\,dt,\quad C_+=2y_b'+\int_0^1 y'(t)\psi'(1-t)\,dt \tag{5.2}$$

Any of the equalities (5.2) may be replaced by the equivalent relation

$$C_+=C_-+2(y_+'-y_-')_d \tag{4'}$$

Equation (4.5) should also be replaced by two equations containing constants C- and C+.

If contour ab does not contain sections of the boundary extremum (2.2), /62
then sets (4.8) remain universal for a given incident flow and fixed values of x_d and y_b/Y, as in the case of a smooth contour.

Let us assume that $x_d = 0.5$ and $y_b = 0$. We shall show that in this case $y'(x)$ and $\mu(x)$ for the optimum contour are odd functions with respect to the point $x = 0.5$, i.e.

$$y'(1-x)=-y'(x),\qquad \mu(1-x)=-\mu(x) \tag{5.3}$$

which, incidentally, gives us $y_b = 0$ when $x_d = 0.5$.

The second equality of (5.2) may be rewritten in the form

$$C_+=2y_b'+\int_0^1 y'(1-t)\psi'(t)\,dt \tag{5.4}$$

Let us assume that the first equality of (5.3) does indeed occur. Then it follows from (5.4) and the first equation of (5.2) that $C_+ = C_-$. Now in order to prove the assumption it is sufficient to show that the substitution $C_+ = -C_-$ in the equation for $y'(1-x)$ and $\mu(1-x)$ leads to (5.3). This is easily done by substituting (4.3) in the specified equations and then using identities (4.4). Finally, the generatrix of the profile $y = y(x)$ appears to be symmetrical with respect to its center in view of (5.3) and (3.4).

6. Computations were carried out for flow with one nonequilibrium process where $\psi(x)$ is given by equation (1.2) while $\psi'(x)$ is

$$\psi'(x) = (\omega - 1)(L^\circ/2K^\circ)\{I_0[(\omega-1)X] - I_1[(\omega-1)X]\}\exp[-(\omega+1)X]$$

In all specified cases $\omega = 2$ while the ratio $L^\circ/K^\circ = 0.1$, 1.0, 10 and 100. We recall that $L^\circ/K^\circ = 0$ corresponds to frozen flow while $L^\circ/K^\circ = \infty$ corresponds to equilibrium flow. In both limiting cases the optimum contour consists of one or several rectilinear segments.

The calculations were made by the iteration method on M-20 electronic computer. When the integrals were computed the ratio $y'(x)/y_a'$ in the integral equations was taken from the results of the preceding approximation. The zero approximation in the case of smooth contour ab was taken as $y'(x)/y_a' \equiv 1$. The zero approximation for a contour with a break was

$$y'(x)/y_a' = \begin{cases} 1 & (0 < x < x_d) \\ (Y - y_b)/(x_d - 1)y_a' & (x_d < x < 1) \end{cases}$$

The range $0 \leqslant x \leqslant 1$ was broken down into 100 equal parts. Calculations with a large number of points showed that this insured accuracy for all signs in the results presented below. The computation of a smooth contour took 2 to 4 minutes while the computation of a contour with a break in the generatrix and with given x_d and y_b/Y required 30 to 60 minutes (the increase in the computation time was due to the necessity of making adjustments for the known value of y_b/Y). In cases when properties of (4.6) or (5.3) are valid, the computation time may be reduced in half. After the contour is constructed c_x is computed and the optimum conditions are checked. These conditions are formulated as inequalities. In particular, when sections (2.2) are present, the inequalities are (3.2).

The flow around contours whose generatrixes pass through the same points a and b or a, b, and d but consist of rectilinear segments was computed for comparison. The value of c_x for segment ab of such contours is designated by C_{xo}.

Some of the parameters of the computed contours are presented in the table. Each line of the table pertains to one optimum contour.

The first three contours are smooth and correspond to different $L°/K°$. The distribution of the slope $\psi(x) \equiv y'(x)/y_a'$ for these as well as for the smooth optimum contour are shown for $L°/K° = 100$ in Figure 4 by solid lines. Only half of the contour is shown because of the properties of summetry (the numbers above the curves give the values of $L°/K°$). Mechanisms can be seen by which the optimum contours for $0 < L°/K° < \infty$ degenerate into rectilinear contours in the limiting cases. When $L°/K°$ decreases we have $\phi(x) \to 1$, i.e., a straight line is approached due to the ever decreasing variation in the slope of the generatrix

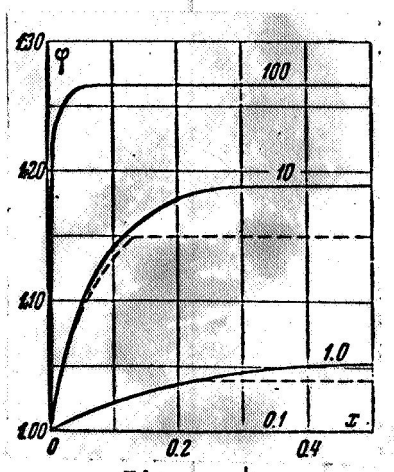

Figure 4.

When L!/K° increases the function $\phi(x)$ tends to become constant which differs /63 from unity everywhere except in small vicinities of point a and b, i.e., the contour becomes straighter over a greater part of segment ab, when $\phi(x)$ varies sharply by a finite amount close to its ends. Two contours ($L°/K°$ - 1.0 and 10) with sections of the boundary extremum (2.2) are shown on the same figure by broken lines. The values of y_a' for these are equal to $\pm$ 0.9615k and $\pm$ 0.8696k respectively. It is interesting to note that outside the sections of the boundary extremum $\phi(x)$ differs little from $\phi(x)$ of contours corresponding to the same values of $L°/K°$ but without sections (2.2). When $y_a'>0$, all given examples yield bodies of the type shown in Figure 1 while for $y_a'<0$ they yield bodies of the type shown in Figure 2.

The last four rows of the table pertain to profiles with breaks at x_d = 0.5. The ratios Y/y_a' for these are 0.4998, 0.4956, 0.5566 and 0.5572, while $y_b/Y = 0$ for the first three and equal to 0.0311 for the last one. The distributions of $\phi(x)$ for the forward part of the profiles are shown in Figure 5 by solid lines (the numbers 1, 2, 3, 4 above them are the numbers of these profiles in the order in which they are tabulated). The broken line shows

$[-y'(x-0.5)/y_a']$ on the tail part of the last profile (for $y_b/Y = 0$ the distribution of $\phi(x)$ on the tail part of the profile is obtained from the condition that it is odd with respect to its center).

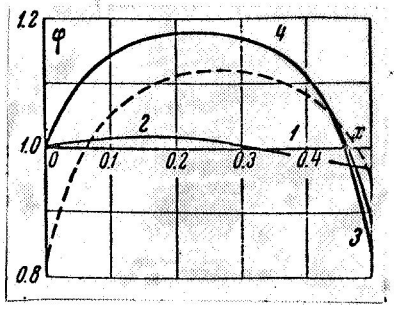

Figure 5.

$L°/K°$	v_b/v'_a	$\beta_{co}(p_{co} - p^+)/v'_a$	c_x/v'^2_a	c_x/c_{x_0}
0.1	1.0007	—1.9541	0.9777	0.99999
1.0	1.0340	—1.7546	0.9072	0.99976
10	1.1626	—1.6811	0.9772	0.99843
0.1	0.0000	1.9980	0.9984	0.99998
1.0	0.0000	1.9037	0.9420	0.99930
10	0.0000	1.6818	0.9336	0.99434
10	0.0311	1.6465	0.8870	0.99726

As we can see from the table the constructed optimum contours yield a reduction in the drag compared with contours made of rectilinear segments, but this reduction is insignificant. This result is valid only for slender profiles when it is meaningful to linearize the equations of kinetics which serve as the basis of the theory. When there are large deviations from equilibrium this linearization becomes invalid due to strong nonlinearity in the right sides of the kinetics equations and a substantial gain is possible here [2].

The authors are grateful to L. Ye. Sternin for his incessant support of this work.

REFERENCES

1. Krayko, A. N., *Variatsionnyye zadachi gazovoy dinamiki nerovnovesnykh i ravnovesnykh techeniy*, [The Variational Problems of Gas Dynamics of Nonequilibrium and Equilibrium Flows], PMM, Vol. 28, No. 2, pp. 285-295, 1964.
2. Galyun, N. S., A. N. Krayko, *Ob odnoy variatsionnoy zadache odomernoy neravnovesnoy gazovoy dinamiki*, [On One Variational Problem of One-Dimensional Nonequilibrium Gas Dynamics], Izv. AN SSSR. MZhG, No. 2, pp. 277-36, 1966.
3. Theory of Optimum Aerodynamic Shapes. Extremal Problems in the Aerodynamics of Supersonic, Hypersonic, and Free-Molecular Flows. New York - London, Acad. Press, 1965.
4. Krayko, A. N., I. N. Naumova, Yu. D. Shmyglevskiy, *K postroyeniyu tel optimal'noy formy v sverkhzvukovom potoke*, [On the Construction of Bodies of Optimum Shape in a Supersonic Flow], PMM, Vol. 28, No. 1, 1964.
5. Krayko, A. N., *Issledovaniye slabo vozmushchennykh sverkhzvukovykh techeniy pri proizvol'nom chisle neravnovesnykh protsessov*, [The Investigation of Weakly Perturbed Ultrasonic Flows with an Arbitrary Number of Nonequilibrium Processes], PMM, Vol. 30, No. 4, pp. 661-673, 1966.

6. Der, J. J., Linearized Supersonic Nonequilibrium Flow Past Arbitrary Boundary. NASA, Technical Report R-119, 1961.
7. Guderley, G., E. Hantsch, *Beste Formen für achsensymmetrische Uberschall-schubdüsen*, Z. Flugwiss, B. 3, H. 9, S. 305-313, 1955.
8. Shmyglevskiy, Yu. D., *Nekotoryye variatsionnyye zadachi gazovoy dinamiki*, [Some Variational Problems of Gas Dynamics], Tr. VTs AN SSSR, 1963.
9. Krayko, A. N., *Variatsionnyye zadachi sverkhzvukovykh techeniy gaza s proizvol'nymi termodinamicheskimi svoystvami*, [Variational Problems of Supersonic Gas Flows with Arbitrary Thermodynamic Properties], Tr. VTs AN SSSR, 1963.

Translated for the National Aeronautics and Space Administration under Contract No. NASw-1695 by Techtran Corporation, P.O. Box 729, Glen Burnie, Maryland 21061

Cover Page Source
NASA